Signal Processing First
Lab 05: Design with Fourier Series – Distortion

Pre-Lab and Warm-Up: You should read at least the Pre-Lab and Warm-up sections of this lab assignment and go over all exercises in the Pre-Lab section before going to your assigned lab session.

Verification: The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing on the **Instructor Verification** line. When you have completed a step that requires verification, simply demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA when you leave the lab.

1 Introduction & Objective

The goal of this laboratory project is to show that Fourier Series analysis is a powerful method for predicting the response of a LTI system when the input is a periodic signal. Since we will be doing Fourier Series for continuous-time signals, the formulas are integrals. As a result, we will use MATLAB's numerical integration capability to calculate the Fourier Series coefficients of the output and the input; this method was introduced in Lab 04. In this particular lab, we will use Fourier Series and the Fourier transform to analyze a frequency synthesis design problem in the frequency domain.

2 Background: Fourier Series Analysis and Synthesis

Recall the *analysis* integral and *synthesis* summation for the Fourier Series expansion of a periodic signal $x(t) = x(t + T_0)$. The Fourier synthesis equation for a periodic signal $x(t) = x(t + T_0)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \tag{1}$$

where $\omega_0 = 2\pi/T_0$ is the *fundamental* frequency. To determine the Fourier series coefficients from a periodic signal, we must evaluate the *analysis* integral for every integer value of k:

$$a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt \tag{2}$$

where $T_0 = 2\pi/\omega_0$ is the *fundamental* period. If necessary, we can evaluate the analysis integral over any period; in (2) the choice $[-1/2\, T_0, 1/2\, T_0]$ is sometimes a convenient one, but integrating over the interval $[0, T_0]$ would also give exactly the same answer.

The Fourier Series representation is extremely useful when studying the effects of an LTI filter, because the output signal is also periodic. The Fourier Series coefficients of the output signal $\{b_k\}$ are obtained by **multiplying** by the frequency response:

$$b_k = a_k\, H(j\omega_0 k) \tag{3}$$

where $H(j\omega_0 k)$ is the frequency response of the LTI system evaluated at the harmonics.

3 Pre-Lab

Review your work in Lab 04. You will need the function `syn_fourier( )` from that lab, and you will need to know how to use numerical integration to evaluate the Fourier analysis integral.

4 Warmup

In this project, we will use the Fourier Series coefficients to predict the response of a LTI system. Since you have already developed the capability to produce the Fourier Series numerically (in Lab 04), the primary point of the warm-up is to show that you can adapt your existing MATLAB functions to do a new example quickly.

4.1 Square Wave Analysis

Evaluate the Fourier Series coefficients for the following periodic square-wave signal which is defined over one period to be:

$$x(t) = \begin{cases} A & \text{for } 0 \le t \le 0.2T_0 \\ 0 & \text{for } 0.2T_0 < t < T_0 \end{cases} \tag{4}$$

(a) Write a MATLAB function that will evaluate the Fourier Series coefficients (see Eq.(2)) for the square wave over the range of indices $k = -N, \ldots, -1, 0, 1, 2, \ldots, N$. The function will contain a `for` loop to do all the coefficients from $k = -N$ to $k = +N$. It should return a vector containing $2N + 1$ elements which are the $\{a_k\}$ coefficients. At this point the function should be general enough to work for any T_0.

(b) Use the Fourier Series function written in part (b) to evaluate the $\{a_k\}$ coefficients for $x(t)$ from Eq. (4) for the case where $A = 3\pi$, and $T_0 = 3.0$ secs. Find the $\{a_k\}$ coefficients for $N = 7$ and make a stem plot of the magnitude of the coefficients versus k.

4.2 Frequency Response of an Analog Filter

In the lab project, you will use a continuous-time LTI system for filtering. In this section of the warm-up, we will investigate the following frequency response:

$$H(j\omega) = \frac{j\omega}{(4-\omega^2)+j\omega/3}$$

(a) In this part, you will have to make a plot of the magnitude and phase of $H(j\omega)$ versus frequency. In order to get values for the plot, you should evaluate the $H(j\omega)$ formula directly for a dense grid of frequencies. Use a range of frequencies that extends from -12 rad/s to $+12$ rad/s. Plot $|H(j\omega)|$ versus ω. What kind of filter is $H(j\omega)$?

(b) Determine the peak value of the magnitude (frequency) response and the location of the peak. Evaluate the frequency response formula to verify that the peak value is correct.

4.3 Filtering a Periodic Signal

In this part, you will have to "filter" the periodic input signal (the square wave from Section 4.1) through a continuous-time LTI system whose frequency response is given in Section 4.2. Since this is an analog system, we cannot do the filtering; instead, we calculate what the output signal would be by finding the Fourier Series of the output.

(a) Determine the Fourier Series coefficients of the output signal $y(t)$ when the input is the periodic square wave defined in (4). Use the frequency response $H(j\omega)$ and apply (3). Make a stem plot of the Fourier coefficients of $y(t)$ versus frequency, over the frequency range -12 rad/s to $+12$ rad/s. Recall that the Fourier coefficients are, in fact, the (magnitude) spectrum of $y(t)$.

(b) Write a MATLAB function that will evaluate signal $y_N(t)$ (see Eq. (1)) over the range of indices $k = -N, \ldots, -1, 0, 1, 2, \ldots, N$. The function will contain a `for` loop to do all the coefficients from $k = -N$ to $k = +N$.

(c) Synthesize an approximation $y_N(t)$ $2N+1=7$ Fourier coefficients